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The effect of mean velocity variations on jet noise

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Experimental evidence suggests that it may be profitable to separate the discussion of the high-frequency components of jet noise from low-frequency components. Theory then leads one to the recognition that the physical mechanism of noise generation is slightly different for the two components and for this reason one may speak of high-frequency 'self noise' and low-frequency 'shear noise' (Lilley 1958; Ribner 1964).

Both theory and experiment indicate that mean-flow refraction effects on self noise are appreciable. Using geometrical acoustics a description of the far-field radiation pattern of the high-frequency end of the jet-noise spectrum is obtained, in good qualitative agreement with observations. A conservation law of acoustic energy, applying between the frame of reference in which the sound sources move, and the fixed frame in which the observations are carried out, results in the absence of the convection amplification effect (over and above the U^8 law) for total sound power at high frequencies, a result which helps explain the uniform validity of the U^8 law.

An analysis of the shear-noise source term suggests that this part of the sound radiation is due to a combination of quadrupoles which have at least one axis parallel to the jet. This then explains the observed concentration of low-frequency noise around the jet axis.

1. Introduction

In the light of detailed knowledge accumulated within the last decade or so on the characteristics of the aerodynamic noise of jets, it would appear to be useful to distinguish between two rather different components of jet noise, characterized in the first instance by different, though overlapping, ranges of frequency.

The two components have a different directivity pattern, the low-frequency noise being concentrated around the jet axis, while the high-frequency noise is more nearly omni-directional, and it seems that the physical mechanism of their generation is also somewhat different. Noise suppressors (the simplest ones being serrations or teeth on a nozzle) reduce the intensity of the low-frequency component but are largely ineffective on the high-frequency noise.

Lilley (1958) has used the classification 'self noise' and 'shear noise' for the higher- and lower-frequency components respectively. The terminology arises from one of Lighthill's (1954) results to the effect that a portion of the noise is directly generated by turbulence (much as would be generated by isotropic turbulence, discussed by Proudman 1953) while another portion is proportional to the mean-flow velocity gradient, the mean 'shear'. Mollö-Christensen, Kolpin & Martucelli (1964) have experimentally demonstrated the different characteristics of the two components of jet noise in a forceful manner. Ribner (1964) has discussed jet noise in some detail on the basis of this classification and has been able to provide a fairly comprehensive and illuminating picture.

Both in Ribner's (1964) review article and Lighthill's (1962) somewhat earlier summary, certain shortcomings of our present understanding of jet noise are conspicuous. Little is known of the effect of mean-flow refraction, although from atmospheric applications it is known that the variation of wind with height can bend sound rays and lead to the formation of 'zones of silence'. For sound of very long wavelength such effects may be expected to be negligible (as has indeed been assumed by Lighthill), but some recent experiments of Atvars, Schubert & Ribner (1965) show beyond doubt that this assumption cannot be made for *all* components of jet noise, even if it is approximately valid at lower frequencies.

A second difficulty concerns the exact physical mechanism of shear-noise emission. Lighthill's (1954) original proposal was than an xy (45°) quadrupole is responsible for this noise, but experiments indicate an xx (0°) quadrupole, as suggested by Ribner (1964), or at least a combination of quadrupoles with one axis along x. Although Ribner has been able to provide theoretical expressions of the correct form to account for shear noise, his calculations are based on an arbitrarily assumed velocity correlation function and a more general discussion of the relevant source terms would seem to be desirable. The understanding of the generation of shear noise is of particular importance since it may shed light on the operation of noise suppressors, which, as remarked before, mainly eliminate low-frequency noise.

The present paper is thus devoted to a discussion of the twin problems of the refraction of 'self noise' by the mean flow and the generation of 'shear noise'.

2. The 'convected' wave equation

In his classic paper on aerodynamic sound generation Lighthill (1952) demonstrated that the equations of continuity and motion may be cast into the form of a wave equation

$$\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \tag{1}$$

where $T_{ij} = \rho v_i v_j + p_{ij} - a_0^2 \rho \delta_{ij}$.

Herein ρ is density, t time, a_0 speed of sound in the undisturbed medium, x_i are space co-ordinates, v_i velocity components and p_{ij} the stress tensor. The left-hand side of this equation describes the propagation of sound in a medium at rest, while the right-hand side may be regarded a collection of all-comprehensive 'source terms', significantly different from zero only in fast-moving regions of the flow and describing a number of different physical effects such as generation, refraction and scattering of sound. Our objective here is to discuss those physical effects which are attributable specifically to the strongly non-uniform mean-velocity field of a jet, and to this end we shall attempt to identify the relevant parts of the source terms.

Using the equation of continuity the leading term in the double divergence of T_{ij} (the momentum flux term) may be expanded, giving the following form of the source terms:

$$S = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} = \rho\left(\frac{\partial v_j}{\partial x_i}\frac{\partial v_i}{\partial x_j} + \frac{\partial v_i}{\partial x_i}\frac{\partial v_j}{\partial x_j}\right) - 2v_i\frac{\partial^2 \rho}{\partial x_i \partial t} - v_i v_j \frac{\partial^2 \rho}{\partial x_i \partial x_j} + \frac{\partial^2}{\partial x_i \partial x_j}(p_{ij} - a_0^2 \rho \delta_{ij}). \tag{2}$$

In the conventional manner of turbulence theory let the velocity-field be now divided into a 'mean' and a 'fluctuating' part:

$$v_i = V_i + v_i'.$$

The above expression then becomes

$$S = \rho\left(\frac{\partial V_j}{\partial x_i}\frac{\partial V_i}{\partial x_j} + \frac{\partial V_i}{\partial x_i}\frac{\partial V_j}{\partial x_j}\right) - 2V_i\frac{\partial^2 \rho}{\partial x_i \partial t} - V_i V_j \frac{\partial^2 \rho}{\partial x_i \partial x_j} + 2\frac{\partial V_i}{\partial x_j}\frac{\partial(\rho v_j')}{\partial x_i} + 2\frac{\partial}{\partial x_j}\left(\rho v_j' \frac{\partial V_i}{\partial x_i}\right) + \frac{\partial^2}{\partial x_i \partial x_j}(\rho v_i' v_j' + p_{ij} - a_0^2 \rho \delta_{ij}). \tag{3}$$

A number of terms in this expression do not depend on the turbulent velocities, only on the mean flow, and contain at the same time, density derivatives. Such terms thus express the interaction of sound waves with the mean velocity field; they are more likely to have relevance to the propagation of sound than to its generation. For this reason the non-turbulent terms in S may be conveniently taken over to the left-hand side of (1):

$$\frac{\partial^2 \rho}{\partial t^2} + 2V_i \frac{\partial^2 \rho}{\partial t\, \partial x_i} + V_i V_j \frac{\partial^2 \rho}{\partial x_i \partial x_j} - a_0^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} - \rho\left(\frac{\partial V_j}{\partial x_i}\frac{\partial V_i}{\partial x_j} + \frac{\partial V_i}{\partial x_i}\frac{\partial V_j}{\partial x_j}\right) = Q, \tag{4}$$

where the new 'source term' is given by

$$Q = \frac{\partial^2}{\partial x_i \partial x_j}(\rho v_i' v_j' + p_{ij} - a_0^2 \rho \delta_{ij}) + 2\frac{\partial V_i}{\partial x_j}\frac{\partial(\rho v_j')}{\partial x_i} + 2\frac{\partial}{\partial x_j}\left(\rho v_j' \frac{\partial V_i}{\partial x_i}\right). \tag{5}$$

The main difference between (1) and (4) is that differentiation following the mean motion, $(\partial/\partial t + V_i \partial/\partial x_i)$, replaces $\partial/\partial t$. This evidently describes the *convection* of sound by the mean flow. The last term on the left of (4), containing the density, may be shown to be of a smaller order of magnitude in jets than the remainder by a boundary-layer-type argument, and we shall not be concerned with it here. Without this term the equation is identical with that describing the propagation of sound in a uniformly moving medium (Blokhintsev 1946).

An important point, however, is that mean velocity gradients appear in the source term (5). We shall return to these terms after having discussed convection effects.

The principal physical effects of sound convection are visualized with relative ease for the two extreme cases of very short or very long wavelength of the radiated sound (short or long compared with the thickness of the shear layer). Sound of very short wavelength is known to suffer bending of the rays or 'refraction', as in the atmosphere where the wind velocity increases with height. This leads to the formation of 'zones of silence' discussed by Lord Rayleigh. The behaviour of such short-wavelength sound may be described by ray theory or geometrical acoustics.

On the other hand, when the wavelength of the radiated sound is long compared with the thickness of the shear layers any density excess or deficiency appearing at the source-eddy extends *in the same phase* to a relatively large portion of the fluid, much of it completely outside the jet. The fact that the shape of this region of more or less coherent initial density disturbance is distorted by the mean flow is not likely to be significant: the undisturbed fluid just outside the jet receives the 'signature' of the eddy's pulsations exactly as they occur at the source. In this case it seems to be legitimate to ignore the convection effects as such and replace the source-eddy by a source moving through a medium at rest, as has been done by Lighthill (1952).

Experimental evidence shows that jets radiate sound of a reasonably broad frequency spectrum. At the peak of the spectrum the frequency is such that the long-wavelength approximation appears reasonable. Consequently, the moving-source model may be used in deducing the directivity distribution for most of the radiated sound. However, there are also high-frequency components in the observed noise experimentally easily separated from the rest. In understanding the behaviour of these components a discussion of the short-wavelength approximation may be useful, which is therefore the subject of the next section.

3. Geometrical acoustics of an 'idealized' mixing layer

It is now known that most of the noise of jets originates in the centre portion of the mixing layer, close to the nozzle exit. Here the flow is very nearly parallel and the radial velocity gradient dominant. In order to simplify the theory we shall deal with an 'idealized mixing layer' characterized by

$$\left.\begin{aligned} V_1 = U = f(x_2), \quad V_2 = V_3 = 0,\\ \partial V_1/\partial x_1 = \partial V_1/\partial x_3 = 0. \end{aligned}\right\} \tag{6}$$

These assumptions retain the most essential features of the velocity field while reducing the number of terms in (4) to a level where the theoretical arguments are easily appreciated. It will also be convenient in this section to adopt an (x, y, z) notation. For the idealized mixing layer defined by (6), equation (4) becomes then

$$\frac{\partial^2 \rho}{\partial t^2} + 2U \frac{\partial^2 \rho}{\partial x \partial t} - (a_0^2 - U^2) \frac{\partial^2 \rho}{\partial x^2} - a_0^2 \frac{\partial^2 \rho}{\partial y^2} - a_0^2 \frac{\partial^2 \rho}{\partial z^2} = Q. \tag{7}$$

This 'convected wave equation' is very similar to the equation describing the propagation of sound in a homogeneous moving medium of uniform velocity U, the difference being that in our 'idealized mixing layer' the velocity U is a function of the y co-ordinate.

The principles of geometrical acoustics have been set forth in the exhaustive study of Blokhintsev (1946), for example. The geometry of the sound rays may be obtained by a study of the characteristics of equation (7). For the case of our idealized mixing layer, $U = U(y)$, if ζ is the angle included by a sound ray and the x-axis, the progress of a single ray is described by the simple relationship

$$\cos\zeta/(1+M\cos\zeta) = \cos\zeta_0 = \text{const.}, \tag{8}$$

where $M = U/a$ is the local Mach number and ζ_0 is the angle of emergence of the ray into the undisturbed fluid where $M = 0$. For our purposes here it is permissible to neglect temperature differences, so that $a = a_0$ throughout.

For forward emission the minimum of ζ at the source is 0, giving $\cos\zeta = 1$. This corresponds to a minimum angle of emergence

$$\zeta_{0m} = \cos^{-1}(1+M_s)^{-1} \tag{8a}$$

if M_s is the source Mach number. Conversely, for rearward emission the maximum angle of emergence is $\zeta_0 = \pi$ and the corresponding maximum angle of emission for waves that do escape through the mixing layer is

$$\zeta_{sm} = \cos^{-1}(-1-M)^{-1}. \tag{8b}$$

Consequently, rays emitted directly downstream are deflected and emerge into the undisturbed medium at an angle ζ_{0m}, so that a cone of half angle ζ_{0m} downstream of the source is not reached by any sound rays at all, it becomes a 'cone of silence'. Upstream of the source all space is reached by waves, but much of the emission is trapped inside the mixing layers and, in the case of propulsion jets, disappears into the engine.

The sharp division at the boundary of the forward cone of half angle ζ_{0m} cannot of course exist in reality, not even on the linear theory. As Friedlander (1958) points out, the geometrical-acoustics approximation may be regarded as the first term in a series expansion either in terms of inverse powers of the wave-number (as Blokhintsev has approached the problem) or, for a sound pulse, of time counted from the arrival of the pulse. Thus there will be a considerable 'leakage' by diffraction into the cone of silence. Nevertheless, a sharp peak is to be expected at the angle given by equation (8a).

The intensity of the sound may be calculated from the energy-flux conservation law (Blokhintsev 1946, p. 41)

$$(\overline{p^2}/\rho a^2)(1+M\cos\zeta)V_r\,dS = \text{const.} \quad \text{(along a ray-tube)}, \tag{9}$$

where p is acoustic pressure fluctuation (not absolute pressure), $\overline{p^2}$ being its mean-square value over a sufficiently long period. V_r is the magnitude of the 'ray velocity',

$$\mathbf{V}_r = \mathbf{U} + a\mathbf{n}, \tag{10}$$

with $\mathbf{n}$ = a vector normal to the wave-front; dS is the cross-section of a given individual 'ray tube'. The projection of the ray velocity to $\mathbf{n}$ is the 'phase velocity' V_f. With the section dS_f of the ray tube traced out *on the wave-front* one has the relationship (figure 1)

$$V_r\,dS = V_f\,dS_f. \tag{11}$$

In order to predict the far field of sound radiation by equation (9) it is necessary to know $\overline{p^2}$ somewhere along the ray tubes. What one is looking for is a relationship between source strength Q and $\overline{p^2}$.

A calculation of $\overline{p^2}$ close to the source is possible in a frame of reference attached to the moving source. There are actually two reasons for the use of a moving frame of reference: one, that retarded time effects may in such a frame be expressed with relative ease; two, that one needs a simpler equation than (7) for writing down an explicit solution. In order to carry out the transformation it is necessary to assume that *both* the source-eddy size and the wavelength of the radiated sound are small compared with the thickness of the shear layer. In that case the source element and the first few wavelengths of the radiated sound may be regarded as embedded in fluid travelling at sensibly constant mean velocity $U = M_s a$. Applying the transformation

$$t' = t, \quad x' = x - Ut, \quad y' = y, \quad z' = z \tag{12}$$

to equation (7) one obtains

$$(\partial^2 \rho / \partial t'^2) - a_0^2 \nabla'^2 \rho = Q', \tag{13}$$

where ∇'^2 is the Laplacian, Q' the source term (equation (5)) in moving co-ordinates. It may be remarked here that to assume a source volume to be small compared with the shear-layer thickness is a relatively crude step. It is, however, also made in the moving-source model and there is no reason to doubt its usefulness.

Equation (13) is now the ordinary wave equation and the solution may be written down at once. Let a source element of strength Q' be located at a point of radius vector $\mathbf{s}'$ in the moving frame. The sound pressure at another point, of radius vector $\mathbf{r}'$, is given by

$$dp = \frac{1}{4\pi |\mathbf{r}' - \mathbf{s}'|} [Q'] \, d\tau', \tag{14}$$

where $d\tau' = dx'\,dy'\,dz'$ is the volume element at the source point and the square brackets indicate that Q' is to be evaluated at the retarded time, $t - (|\mathbf{r}' - \mathbf{s}'|/a)$. This solution is valid in the 'far field', i.e. provided that $|\mathbf{r}' - \mathbf{s}'|$ is greater than a wavelength or so. Since we are dealing with the short-wavelength approximation, (14) is appropriate. Integrating over a whole 'eddy volume' or 'correlation volume' the mean-square sound pressure at radius vector r' may be found

$$\delta \overline{p^2} = \frac{\overline{Q'^2}}{16\pi^2 |\mathbf{r}' - \mathbf{s}'|^2} L^3 \, d\tau', \tag{15}$$

where the 'correlation volume' L^3 is defined by the equation

$$\overline{Q'^2} L^3 = \iiint_{-\infty}^{\infty} \overline{Q'(\mathbf{s}) \, Q'(\mathbf{s}^*)} \, d\tau^*. \tag{16}$$

The square brackets have now been dropped on the understanding that retarded time effects over an eddy volume are negligible. When the source term Q is a single or double space derivative, the effect of retarded time is to produce the characteristic pattern of dipole or quadrupole radiation and replace the space by a time derivative, as discussed in detail by Lighthill (1952). Recent experimental evidence (see e.g. Wills 1964) has indicated that eddy convection velocities are not quite identical with mean velocities. Since our moving frame moves with the local mean velocity, significant differences between this and the eddy convection velocity would necessitate taking into account retarded time in a more complicated way. However, in the centre of the mixing layer where most of the noise sources are located the differences are small and do not warrant further consideration.

The task of calculating $\overline{p^2}$ close to the source is now accomplished, except for a return to the fixed frame of reference. A point on the 'ray-tube' is fixed in space, while the source elements $d\tau'$ move at a velocity $M_s a$. As discussed by Lighthill (1952) in some detail, an apparent length-contraction takes place due to retarded times

$$d\tau' = d\tau/(1+M_s\cos\zeta). \tag{17}$$

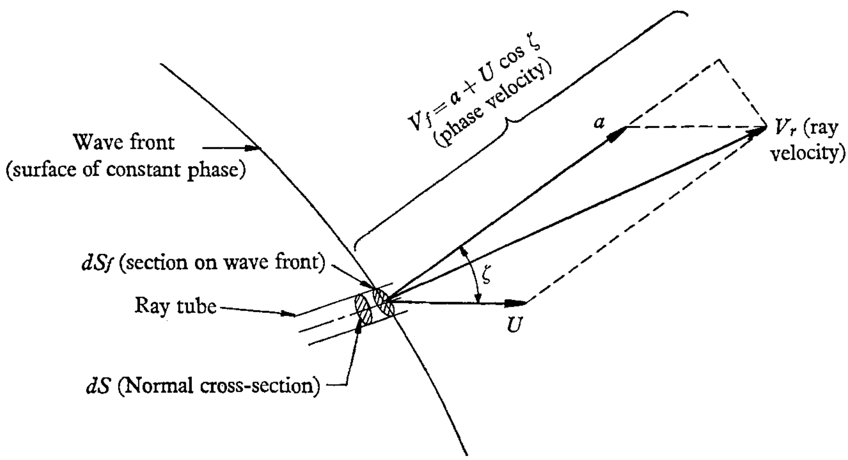

FIGURE 1. Propagation of sound in a moving medium.

The source term Q' we intend to leave in a form to be evaluated in a moving frame. The distance $|\mathbf{r}'-\mathbf{s}'|$ would be inconvenient; we replace it by

$$dS_f = |\mathbf{r}'-\mathbf{s}'|^2 d\Omega,$$

where $d\Omega$ is the solid angle of a ray-tube and dS_f is the cross-section element along a wave-front, as indicated in figure 1. The ray-tube spoken of here is, of course, the one connecting $\mathbf{s}'$ and $\mathbf{r}'$ in the moving frame. Equation (15) thus transforms into

$$\delta\overline{p^2}\,dS_f = \frac{\overline{Q'^2}\,d\Omega}{16\pi^2(1+M_s\cos\zeta)}\,L^3\,d\tau. \tag{18}$$

For this particular ray-tube we may now determine the constant in (9). Making use of (11) and noting that $V_f = a(1+M_s\cos\zeta)$ (see figure 1) one finds

$$\frac{\delta\overline{p^2}}{\rho a^2}(1+M\cos\zeta)V_r\,dS = \frac{L^3\overline{Q'^2}\,d\Omega\,d\tau}{16\pi^2\rho_s a_s}(1+M_s\cos\zeta). \tag{19}$$

The notation '$\delta\overline{p^2}$' is necessary because this is the intensity radiated by an elemental eddy volume, and is proportional to volume. If mean density differences are negligible, we may set here $a = a_s = a_0$, $\rho = \rho_s = \rho_0$; ρ_s, a_s being mean density and speed of sound at the source-volume.

A rectangular element dS_f on the wave-front is described by $d\Omega = d\psi\, d\zeta$. As the ray leaves the high-velocity regions $d\psi$ remains unaffected but $d\zeta$ changes in a manner that can be deduced from (8). At a sufficiently large distance from the source region it is a permissible approximation for practical purposes to write for the cross-section of a ray-tube originating from a region of Mach number M_s (see figure 2)

$$dS = R^2\, d\psi\, d\zeta_0. \tag{20}$$

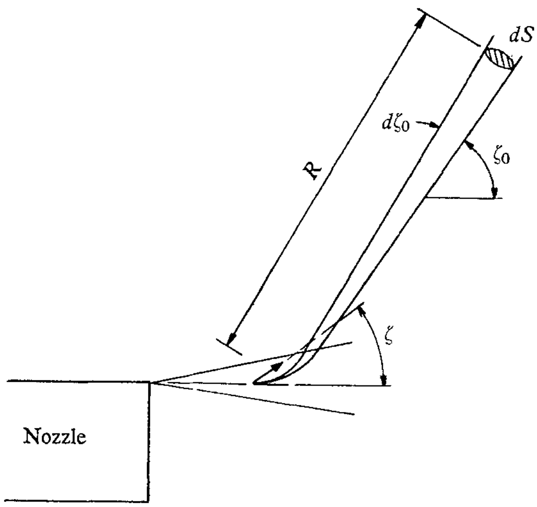

FIGURE 2. Ray tube in the far field.

Thus the intensity of sound in the far field ('far' in the sense that (20) is valid) may be expressed from (19) as

$$\overline{\delta p^2} = \frac{L^3\overline{Q'^2}}{16\pi^2R^2}\frac{d\zeta}{d\zeta_0}(1+M_s\cos\zeta)\,d\tau, \tag{21}$$

where ζ is the angle given by (8) with $M = M_s$.

The last equation describes the directivity pattern of high-frequency radiation emitted by a single 'eddy volume'. The quadrupole nature of the radiation—with its characteristic directivity pattern—is hidden in the source function $\overline{Q'^2}$. Consider, for example, the x–x quadrupole

$$T_{11} = \rho u'^2. \tag{22}$$

The sound intensity field of this particular quadrupole in the *moving frame* is proportional to $\cos^4\zeta$:

$$\overline{Q'^2_{xx}} \sim \frac{1}{a_s^4}\left[\frac{\partial^2(\rho u'^2)}{\partial t^2}\right]^2\cos^4\zeta. \tag{23}$$

Here the usual conversion of a space into a time derivative has been carried out (Lighthill 1952). Far from the source, by equation (21), this yields

$$\overline{\delta p^2} = \frac{1}{a^4}\frac{L^3\overline{[\partial^2(\rho u'^2)/\partial t^2]^2}}{16\pi^2R^2}f(\zeta)\,d\tau, \tag{24}$$

where the 'directivity factor' $f(\zeta)$ is given by

$$f(\zeta) = \frac{d\zeta}{d\zeta_0}\cos^4\zeta(1+M_s\cos\zeta). \tag{25}$$

By equation (8) the directivity factor may be expressed as a function of the far-field angle ζ_0:

$$f = \frac{\cos^4\zeta_0}{(1-M_s\cos\zeta_0)^5}\frac{d\zeta}{d\zeta_0}, \tag{26}$$

where, to write the derivative out explicitly

$$\frac{d\zeta}{d\zeta_0} = \frac{\sin\zeta_0}{(1-M_s\cos\zeta_0)\sqrt{\{(1-M_s\cos\zeta_0)^2-\cos^2\zeta_0\}}}. \tag{27}$$

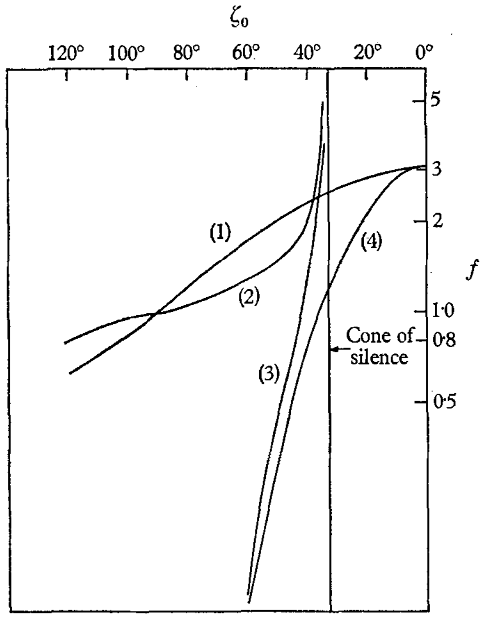

FIGURE 3. Directivity pattern of radiation at source Mach number $M_s = 0{\cdot}2$. The curves show: (1) Long-wavelength extreme (moving-source model), source 3 equal and mutually perpendicular quadrupoles (which in stagnant medium would radiate at equal intensity in all directions); (2) short-wavelength extreme (geometrical acoustics), source as for curve (1); (3) short-wavelength extreme, source a quadrupole with both its axes parallel to the x-axis ($\zeta_0 = 0$); (4) long-wavelength extreme, source as for curve (3).

The directivity factor of (26) strongly resembles Lighthill's (1952) corresponding result for long wavelength (as corrected by Williams 1963 to the 5th power, instead of the 6th, of $1-M_s\cos\zeta_0$), the difference being the $d\zeta/d\zeta_0$ factor. This is true for an x–x quadrupole; for a combination of quadrupoles such that the radiation at the source is *omni-directional* (a case relevant to isotropic turbulence, see Proudman 1953) geometrical acoustics yield the completely different directivity factor (equation (25) without the $\cos^4\zeta$ term)

$$f = \frac{1}{1-M_s\cos\zeta_0}\frac{d\zeta}{d\zeta_0}. \tag{28}$$

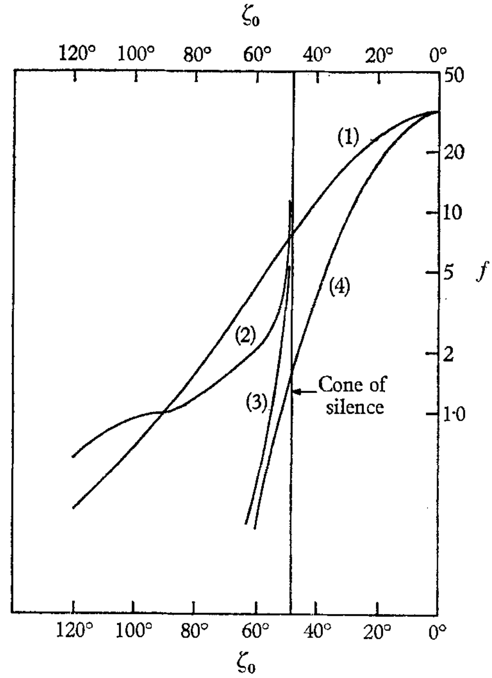

FIGURE 4. As figure 3, for source Mach number $M_s = 0{\cdot}5$.

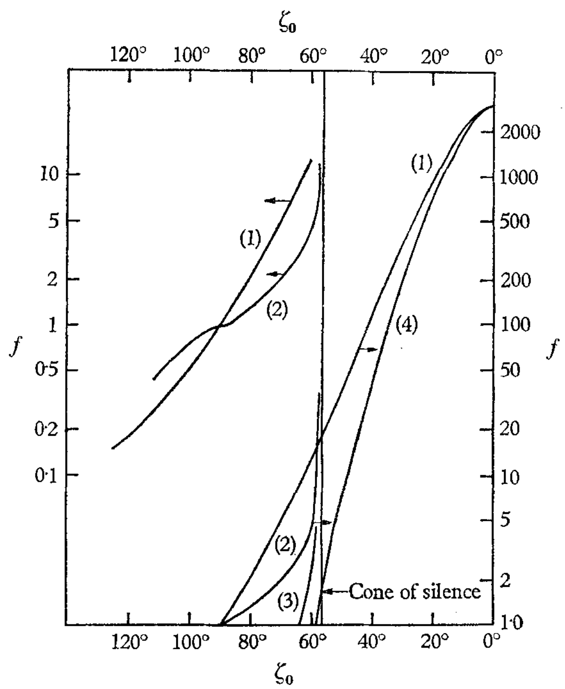

FIGURE 5. As figure 3, for source Mach number $M_s = 0{\cdot}8$.

Figures (3) to (5) illustrate the directivity distributions of the short- and long-wavelength extremes for an x–x quadrupole and omni-directional radiation respectively. The sharp cut-off at the edge of the cone of silence is of course unrealistic, being a defect of the geometrical acoustics approximation, as pointed out before.

It is of interest to note that for the short-wavelength extreme an energy-conservation law holds for the far-field sound radiation: integrating equation (21) over all angles ζ_0, one finds

$$\int_{-\pi}^{\pi} \overline{\delta p^2}\, d\zeta_0 = \int_{\zeta=-\pi}^{\zeta=\pi} \frac{L^3 \overline{Q'^2}}{16\pi^2 R^2} (1 + M_s \cos\zeta)\, dy\, d\zeta, \tag{29}$$

which precisely equals the integral of sound emission as evaluated in the moving frame (provided that $\overline{Q'^2}$ contains only even powers of $\cos\zeta$ and $\sin\zeta$, but this is certainly true of quadrupole radiation). Thus the extra Mach-number dependence (over and above the 8th power law) following from the moving-source model does not apply to high-frequency sound, a conclusion already reached by Ribner (1958) on the basis of considerations akin to geometrical acoustics.

4. The structure of jet noise

We now return to the right-hand side of (4), expressed in (5). For our idealized mixing layer, expressed in the moving frame of reference, this source term becomes

$$Q' = \frac{\partial^2}{\partial x_i' \partial x_j'} (\rho v_i' v_j' + p_{ij} - a_0^2 \rho \delta_{ij}) + 2 \frac{\partial(\rho v_2')}{\partial x_1'} \frac{\partial U}{\partial x_2'} - 2 v_i' U \frac{\partial^2 \rho}{\partial x_i' \partial x_1'}. \tag{30}$$

If one regards ρ as a variable, this source term expresses 'turbulence-sound' interaction, as well as sound generation. The former effects are irrelevant for the present discussion, since as shown by Lighthill (1953), scattering of the sound by the same eddies that generate it is more or less negligible. To exclude such scattering effects one may set $\rho = \rho_0 = \text{const}$. Of the remainder, the term involving $p_{ij} - a_0^2 \rho \delta_{ij}$ depends on temperature differences with which we shall not be concerned. One of the remaining two terms

$$Q_t' = \rho_0 \frac{\partial^2 (v_i' v_j')}{\partial x_i' \partial x_j'} \tag{31}$$

expresses the contribution of the turbulent velocity fluctuations to noise generation. In Lilley's (1958) terminology this is responsible for 'self-noise', while 'shear-noise' is produced by the other remaining term

$$Q_s' = 2\rho_0 \frac{\partial v_2'}{\partial x_1'} \frac{\partial U}{\partial x_2'}. \tag{32}$$

This source term, although identified by Mollö-Christensen & Narasimha (1960), has not been adequately investigated before. Its appearance is that of a dipole source term, since the only time-varying part is $\partial v_2'/\partial x_1'$, which is a simple space-derivative rather than a double one. The mean velocity gradient $\partial U/\partial x_2'$ may be regarded as constant over an eddy volume. In an idealized mixing layer it is also equal to the velocity gradient measured in a fixed frame. Thus the effect of 'retarded time' in the solution (14) (in the moving frame) is merely to replace a space- by a time-gradient

$$dp = \frac{r_1' - s_1'}{2\pi a\,|\mathbf{r}' - \mathbf{s}'|^2}\frac{\partial U}{\partial x_2}\frac{\partial(\rho_0 v_2')}{\partial t}\,d\tau. \tag{33}$$

Note that in the long-wavelength approximation discussed by Lighthill (1952) the same source term arises, the difference in directivity distributions entering when one transfers from the moving frame to the fixed frame. Thus the source term (32) and its expression in terms of a time-derivative in (33) is generally valid.

The quadrupole nature of the apparent dipole (32) may now be recognized. With $\partial U/\partial x_2$ sensibly constant over a source volume one may write the equation of motion

$$\frac{\partial(\rho_0 v_2')}{\partial t} = -\frac{\partial}{\partial x_i'}(\rho_0 v_2' v_i' - p_{2i}). \tag{34}$$

This being a divergence, the space-derivatives may again be replaced by time derivatives, giving, by (33),

$$dp = -\frac{(r_1' - s_1')\,(r_i' - s_i')}{2\pi a_s^2\,|\mathbf{r}' - \mathbf{s}'|^3}\frac{\partial U}{\partial x_2}\frac{\partial(\rho_0 v_2' v_i' - p_{2i})}{\partial t}\,d\tau. \tag{35}$$

This is now a quadrupole; it is proportional to the first time-derivative of the Reynolds stress $\rho_0 v_2' v_i'$, whereas the self-noise source terms yield an amplitude dp proportional to the second time-derivative of $\rho_0 v_i' v_j'$. With $i = 2$ (35) gives a contribution proportional to $\partial p/\partial t$, a source term identified by Lighthill (1954). The latter is an x–y quadrupole; (35) shows that the shear-noise source term contains also x–x and x–z quadrupoles.

Thus the physical differences between 'self-noise' and 'shear-noise' may be summed up as follows:

(1) Shear-noise amplitude is proportional to the first, rather than the second, time-derivative of the Reynolds stress, and is therefore likely to radiate at lower characteristic frequencies than self-noise.

(2) Of all three shear-noise quadrupoles at least one axis coincides with the x-axis so that a predominantly forward emission is inevitable, as against self-noise which contains all quadrupoles and may presumably be received in all directions; however, shear noise is not necessarily an x–y quadrupole only, as would follow from Lighthill (1954).

The net outcome of the theoretical investigation is that low frequencies are likely to be concentrated around the x-axis.

5. Discussion

The main theoretical conclusions arrived at above may now be compared with some experimental evidence. It should be emphasized, however, that the geometrical acoustics approximation is based on some rather drastic simplifying assumptions and qualitative agreement is the most to be hoped for. Furthermore, little is known of the exact source terms, which are fourth-order correlations involving Reynolds-stress time-derivatives, and nothing as complicated as this has ever been measured. Thus one is restricted to considering principal characteristics of jet noise and major trends in any changes of those characteristics.

Refraction effects may be expected to appear at relatively high frequencies, perhaps above $fD/U_0 = 2$. The wavelength of sound of this Strouhal number is $\lambda = a_0/f = \frac{1}{2}M_0 D$, which is about equal to the thickness of the shear layer a few diameters downstream of the orifice. Mollö-Christensen *et al.* (1964) have measured the radiation field of such high-frequency sound, separately from the complete sound output. For a 1 in. diameter jet at a Mach number $M_0 = 0{\cdot}8$ the peak sound intensity occurred (their figure 7) at 48°, although in a 'separate experiment' their graph (their figure 11) shows this peak at 41°.

Jet Mach no., M_0		0·6	0·8	0·9
$M_s = 0{\cdot}63M_0$		0·378	0·504	0·567
Angle of peak radiation	Mollö-Christensen *et al.* figure 7	—	48°	—
ζ_{0m}	Mollö-Christensen *et al.* figure 11	38°	41°	41°
	Geometrical acoustics	43°	48°	50°
	Moving-source model	36°	$29\frac{1}{2}$°	28°

TABLE 1. Position of peak high-frequency radiation comparison of theory and experiment

If the geometrical acoustics approximation is valid for sound in this frequency range, the peak should occur at the angle given by equation (8*a*):

$$\zeta_{0m} = \cos^{-1}(1+M_s)^{-1}.$$

It may be assumed that the sound sources are concentrated in that part of the mixing layer where the turbulent intensity is highest (Lilley 1958; Ribner 1964). From the data quoted by Townsend (1956) one may deduce that the highest intensity occurs where $U/U_0 = 0{\cdot}63$, or so. Wills (1964) has directly measured convection velocities U_c of eddies within the mixing layer and has found that, in the area where the turbulent intensity peaks, $U_c = 0{\cdot}63U_0$, and is more or less constant with y (or at least changing more slowly than U). There is therefore ample evidence to set

$$M_s = 0{\cdot}63 \times 0{\cdot}8 = 0{\cdot}504$$

for the case shown in figure 7 of Mollö-Christensen *et al.* Using the moving-source model, Lighthill (1952) explains the occurrence of a peak not far from 45° by the dominance of a T_{12} quadrupole, distorted through source motion. The position of the peak produced by this mechanism may be taken from his figure 3. The comparison between the results of Mollö-Christensen *et al.*, the geometrical acoustics model and the moving-source model is shown in table 1.

This table indicates the basic correctness of the geometrical acoustics approximation for describing the behaviour of high-frequency sound. Further confirmation may be obtained by calculating the ratio of intensities, as expected at $\zeta_0 = 60°$ and 120°, and comparing it with observation. From Lighthill's figure 3 this ratio may be estimated to be 20 at $M_s = 0{\cdot}5$ ($M_0 = 0{\cdot}8$). To apply geometrical acoustics one must decide what quadrupoles dominate self-noise: there are good reasons for expecting this to be omni-directional (Proudman 1953). Then the relevant directivity factor is given by equation (28). The ratio of intensities so calculated is 5·67. The observed value is 3·55, which is thus less than either theoretical value, but the geometrical acoustics approximation gives at least the correct order of magnitude.

As to low-frequency sound, $fD/U_0 < 0{\cdot}5$, the results of Mollö-Christensen *et al.* show the concentration of sound around the x-axis, as has also been recognized by Ribner (1964). Refraction effects in this frequency range seem to be absent or are at least not detectable at 30°. However, it is clear from the data assembled by Howes (1960) that a 'cone of silence' does exist between $\zeta_0 = 0$ and 15° or so for jet Mach numbers of $M_0 = 0{\cdot}6$ and above. The refraction effects are apparently intermediate between zero (moving-source model) and the predictions of geometrical acoustics. It is noteworthy in this context that the Strouhal-number criterion for sound being of 'high frequency' changes with Mach number: with increasing M_0 a greater and greater proportion of the sound would be affected by refraction.

One further point is that the validity of geometrical acoustics for the higher-frequency components helps to explain why the sound radiation does not increase over and above the U^8 power law with increasing Mach number. As pointed out before, an energy conservation law holds as between the moving frame and the fixed frame for high-frequency sound. Moreover, with increasing Mach number this applies to an increasing proportion of the sound, and supplies a further mechanism (apart from the reduction of turbulent intensity with increasing Mach number) for counteracting the increased acoustic energy radiation following from the moving-source model.

Another observed fact is that the peak of the spectrum curve shifts to lower Strouhal numbers with increasing Mach number (Ribner 1964). This is at once easily understood if one accepts that the low-frequency noise is subject to the moving-source amplification discussed by Lighthill (1952), while the high-frequency noise is not. Clearly, under such conditions the dominance of the shear noise would tend to increase with Mach number.

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